

Mark Scheme (Results)

Summer 2017

Pearson Edexcel GCE
Mathematics/Further Mathematics

Decision Mathematics D2 (6690/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1. (a)(i)	Prim's starting from A: AE, AC, BE, AD; EF	M1 A1
	$2 \times 391 = 782$	B1
(ii)	Nearest neighbour: A – E – B – C – D – F – A 69 77 94 97 125 97 = 559	M1 A1 **(5)**
(b)	$500 \leq$ length ≤ 559 (accept $500 <$ length ≤ 559)	B2, 1, 0 **(2)**
		7 marks

Notes for Question 1

a1M1: First four arcs (or first five nodes: A, E, C, B, D or equivalent numbers across the top of the table {1, 4, 3, 5, 2, -}) selected correctly. Award M1 only for a correct tree with no working or for a correct tree starting at a different node
a1A1: CAO (order of arcs correct or all six nodes correct: A, E, C, B, D, F – but not just the numbers across the top of the table)
a1B1: CAO (782) – must follow from the correct MST (so dependent on at least the M mark in (a)(i)) – do not isw if attempt at short cuts reduces this value
a2M1: Nearest neighbour A – E – B – C – D – F – (condone lack of return to start) **or** correct route length of 559. Accept AE, EB, BC, CD, DF but do not accept weights only
a2A1: CAO both route (either in terms of vertices or arcs but not weights) **and** length correct

b1B1: Any indication of an interval from 500 to their 559 (their 559 > 500 and is the smallest value from either the MST method or NN method – must have stated two values in (a) but ignore how these values were derived)
b2B1: $500 \leq$ length ≤ 559 **or** $500 <$ length ≤ 559 (no ft on this mark) – accept set notation e.g. [500,559] or (500, 559]

Question Number	Scheme	Marks
2. (a)		B1 **(1)**
(b)	giving	M1 A1 **(2)**
(c)	giving **Entering cell B1, Exiting cell C4**	M1 A1 M1 A1 **(4)**
(d)	Not optimal as there is a negative improvement index (A3)	M1 A1 A1 **(3)**
		10 marks

	1	2	3	4	Supply
A	21	12			33
B		5	16		21
C			12	13	25
Demand	21	17	28	13	

	1	2	3	4
A	(21)	$12-\theta$		θ
B		$5+\theta$	$16-\theta$	
C			$12+\theta$	$13-\theta$

	1	2	3	4
A	21			12
B		17	4	
C			24	1

Shadow costs		15	-2	5	11
		1	2	3	4
0	A	X	19	15	X
13	B	-16	X	X	-3
5	C	-2	10	X	X

	1	2	3	4
A	$21-\theta$			$12+\theta$
B	θ	(17)	$4-\theta$	
C			$24+\theta$	$1-\theta$

	1	2	3	4
A	20			13
B	1	17	3	
C			25	

Shadow costs		15	14	21	11
		1	2	3	4
0	A	X	3	-1	X
-3	B	X	X	X	13
-11	C	14	10	X	16

Question Number	Scheme	Marks

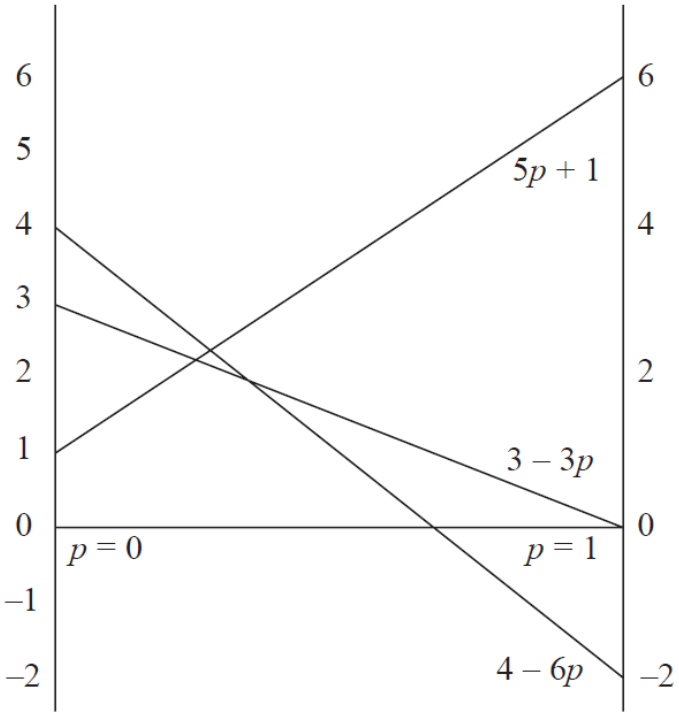
Notes for Question 2

a1B1: CAO

b1M1: A valid route, only one empty square A4 used, θ's balance – some candidates are verifying that A4 is the entering cell (which is fine). For those that start at an incorrect entering cell then the M marks only are available in subsequent parts (unless recovered to the answers given in the scheme)
b1A1: Correct route, up to an improved solution (six numbers no zeros) – if there is a zero in cell A2 then A0 unless corrected in part (b)

c1M1: Finding 7 shadow costs and 6 improvement indices
c1A1: Shadow costs [Alt: A(15), B(28), C(20), 1(0), 2(-17), 3(-10), 4(-4)] and improvement indices CAO
c2M1: A valid route, their most negative II chosen, only one empty square used, θ's balance
c2A1: CSO (for part (c)) – so all previous marks in this part must have been awarded – **including exiting and entering cells stated correctly** (entering is B1 and exiting is C4) – six numbers no zeros

d1M1: Finding 7 shadow costs **and** all 6 IIs **or** sufficient number of shadow costs for at least 1 negative II found
d1A1: CAO A3 = -1 as an II from correct working
d2A1: CSO (for part (d)) + not optimal + reason [Alt shadow costs: A(15), B(12), C(4), 1(0), 2(-1), 3(6), 4(-4)]

Question Number	Scheme	Marks
3. (a)	Row minima: -2, 1, -3 max is 1 Column maxima: 3, 4, 6 min is 3 Play safe is A plays 2 and B plays 1	M1 A1 A1 **(3)**
(b)	Row maximin (1) $\neq$ Column minimax (3) so not stable	B1 **(1)**
(c)	Row 2 dominates row 3 so delete row 3	B1
	Let A play 1 with probability p and 2 with probability $1-p$	
	If B plays 1 A's expected winnings are $3(1-p)=-3p+3$ If B plays 2 A's expected winnings are $-2p+4(1-p)=-6p+4$ If B plays 3 A's expected winnings are $6p+(1-p)=5p+1$	M1 A1
		M1 A1
	$5p+1=3-3p \Rightarrow p=\frac{1}{4}$	DM1 A1
	A should play row 1 with probability $\frac{1}{4}$, row 2 with probability $\frac{3}{4}$ and row 3 never	A1 **(8)**
(d)	Value of the game to player B is $-\frac{9}{4}$	B1 **(1)**
		13 marks

Question Number	Scheme	Marks

Notes for Question 3

a1M1: Clear attempt to find the Row maximin and Column minimax (either the Row minimums or Column maximums correct **or** at least four (of the six) values stated correctly) – some candidates are removing row 3 at this stage which is fine – they will therefore need to find at least four (of the five) correct values for this mark to be awarded
a1A1: Correct Row maximin **and** Column minimax (dependent on all row mins **and** column maxs correct) – these could either be stated or clearly shown
a2A1: Correct play safe for A (2) **and** B (1) – not dependent on the previous A mark

b1DB1: CAO (dependent on all rowmins and colmaxs correct) states $1 \neq 3$ (or row (maximin) $\neq$ col (minimax) as long as 1 is clearly identified as the row maximin and 3 as the column minimax) **and** draws the correct conclusion

c1B1: CAO row 2 dominates row 3 (maybe implied by later working) – accept reduced matrix or 'row 2 dominates row 3' or row 3 crossed out
c1M1: Setting up three probability expressions (allow $p-1$), implicit definition of 'p'
c1A1: CAO (condone incorrect simplification)
c2M1: Attempt at their three lines (correct slant direction and relative intersection with 'axes'), accept $p > 1$ or $p < 0$ here but must go from axis to axis (give bod if close). Must be functions of p
c2A1: CAO $0 \leq p \leq 1$, scaling correct and clear (expect to see 1 line = 1, although other scalings are acceptable eg 1 line = 2), condone lack of labels. Rulers used
c3DM1: Finding their correct optimal point, must have three lines and set up an equation to find $0 \leq p \leq 1$. Dependent on previous M mark. Must have at least three intersection points. Solving all three simultaneous equations and stating incorrect p is M0
c3A1: CAO
c4A1: CSO (must have scored all previous marks in (c)) – all three options listed, check page 1 for A should never play 3

d1B1: CAO

SC1: If row 2 is deleted in (c) candidates can earn a maximum in (c) and (d) of

(c) B0 M1 A0 M1 A0 M1 A0 A1 (d) B1 (max. of 5) – the final A mark is for A should play 2 never, play 1 and 3 with probability $\frac{1}{2}$. The B mark in (d) is for $\frac{1}{2}$
SC2: If row 1 is deleted in (c) candidates can earn a maximum in (c) and (d) of

(c) B0 M1 A0 M1 A0 M0 A0 A0 (d) B0 (max. of 2)

If candidates remove a column then send to review

Question Number	Scheme	Marks
4.		

	1	2	3	4
A	53	84	-	20
B	87	72	41	38
C	70	51	52	25
D	45	-	81	70

	Let x_{ij} be 0 or 1 $\begin{cases}1 \text{ if worker } (i) \text{ does task } (j)\\0 \text{ otherwise}\end{cases}$	B1
	where $i \in \{A,B,C,D\}$ and $j \in \{1,2,3,4\}$	B1
	minimise $C = 53x_{A1} + 84x_{A2} + '200'x_{A3} + 20x_{A4}$ $+87x_{B1} + 72x_{B2} + 41x_{B3} + 38x_{B4}$ $+70x_{C1} + 51x_{C2} + 52x_{C3} + 25x_{C4}$ $+45x_{D1} + '200'x_{D2} + 81x_{D3} + 70x_{D4}$	M1 A1
	subject to $x_{A1} + x_{A2} + x_{A3} + x_{A4} = 1$ or $\sum x_{Aj} = 1$ $x_{B1} + x_{B2} + x_{B3} + x_{B4} = 1$ or $\sum x_{Bj} = 1$ $x_{C1} + x_{C2} + x_{C3} + x_{C4} = 1$ or $\sum x_{Cj} = 1$ $x_{D1} + x_{D2} + x_{D3} + x_{D4} = 1$ or $\sum x_{Dj} = 1$ $x_{A1} + x_{B1} + x_{C1} + x_{D1} = 1$ or $\sum x_{i1} = 1$ $x_{A2} + x_{B2} + x_{C2} + x_{D2} = 1$ or $\sum x_{i2} = 1$ $x_{A3} + x_{B3} + x_{C3} + x_{D3} = 1$ or $\sum x_{i3} = 1$ $x_{A4} + x_{B4} + x_{C4} + x_{D4} = 1$ or $\sum x_{i4} = 1$	M1 A1 A1
		7 marks

Notes for Question 4

1B1: Possible values of x_{ij} (not just *x*) defined. Must be clear that x_{ij} can take only the two values of 0 and 1 and 1 must be attributed to the worker doing the task (*i* and *j* do not need to be mentioned here) and 0 otherwise

2B1: Defining the set of values for *i* and *j* – { } not required – this mark is not dependent on the first B mark

1M1: Attempt at a '16' term expression, coefficients 'correct', 2 'large' values (must be at least 88) included, condone 2 slips (a slip here is an *x* missing from a term, an incorrect coefficient, *ij* confused in a single term or a missing/extra term)

1A1: CAO + minimise

2M1: Four equations with four variable terms, unit coefficients, = 1, allow *x* missing and *ij* confused but not using x_{11} etc.

2A1: Any four equations CAO

3A1: All eight equations **only** CAO (ignore mention of $x_{ij} \geq 0$)

Question Number	Scheme	Marks
5. (a)(i)	$P = x + 2y + 5z$	B1
(ii)	$15x - 2y + 3z \le 180$ $10x + y + z \le 80$ $x + 6y - 2z \le 100$	M1 A1 **(3)**
(b)		

b.v	x	y	z	r	s	t	value
r	15	-2	**3**	1	0	0	180
s	10	1	1	0	1	0	80
t	1	6	-2	0	0	1	100
P	-1	-2	-5	0	0	0	0

b.v	x	y	z	r	s	t	value	row ops
z	5	$-\frac{2}{3}$	1	$\frac{1}{3}$	0	0	60	$R_1 \div 3$
s	5	$\mathbf{\frac{5}{3}}$	0	$-\frac{1}{3}$	1	0	20	$R_2 - R_1$
t	11	$\frac{14}{3}$	0	$\frac{2}{3}$	0	1	220	$R_3 + 2R_1$
P	24	$-\frac{16}{3}$	0	$\frac{5}{3}$	0	0	300	$R_4 + 5R_1$

M1 A1

M1 A1

b.v	x	y	z	r	s	t	value	row ops
z	7	0	1	$\frac{1}{5}$	$\frac{2}{5}$	0	68	$R_1 + \frac{2}{3}R_2$
y	3	1	0	$-\frac{1}{5}$	$\frac{3}{5}$	0	12	$R_2 \div \frac{5}{3}$
t	-3	0	0	$\frac{8}{5}$	$-\frac{14}{5}$	1	164	$R_3 - \frac{14}{3}R_2$
P	40	0	0	$\frac{3}{5}$	$\frac{16}{5}$	0	364	$R_4 + \frac{16}{3}R_2$

M1 A1ft

M1 A1 **(8)**

(c)	$P = 364;\ x = 0;\ y = 12;\ z = 68;\ r = s = 0;\ t = 164$	M1 A1 **(2)**
		13 marks

Question Number	Scheme	Marks

Notes for Question 5

ai1B1: CAO - allow in any equivalent form e.g. $P-x-2y-5z=0$ but not say $P=x+2y+5z=0$
aii1M1: Two inequalities (or equations with slack variables) correct
aii1A1: CAO

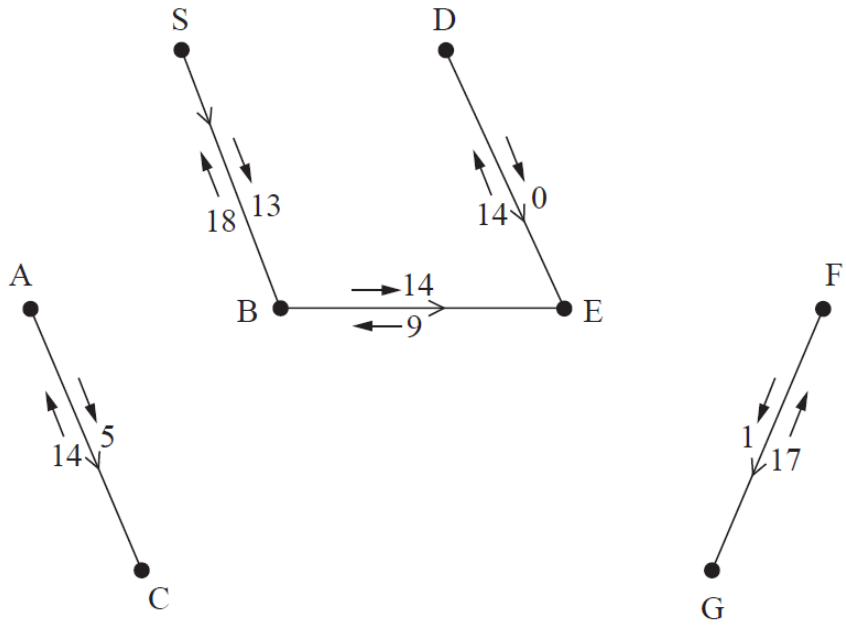
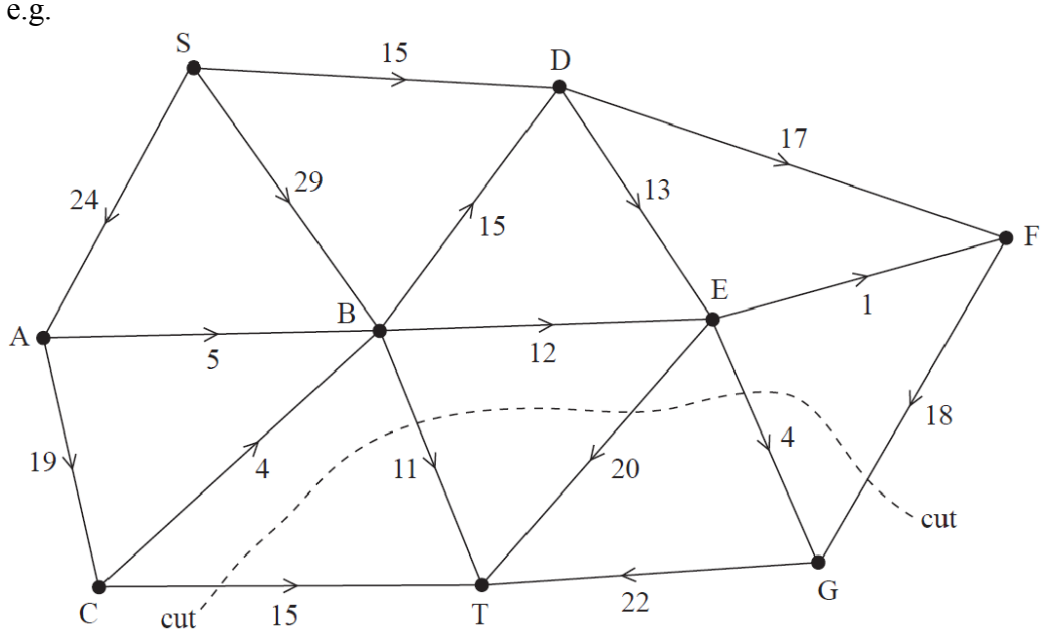
b1M1: Correct pivot located (3 in the *z* column), attempt to divide row. If choosing negative pivot then M0M0
b1A1: CAO pivot row correct including change of b.v. (so *r* must be changed to *z*)
b2M1: (ft) **All** values in one of the non-pivot rows correct **or** one of the non zero/one columns (*x, y, r* or value) correct following through their choice of pivot
b2A1: CAO on all values for the first iteration – ignore row ops and b.v. column for this mark

b3M1: Their correct pivot located following their first iteration, attempt to divide row. If choosing negative pivot M0M0 - however, allow recovery for the third and fourth M marks only if positive pivot chosen for the second iteration after a negative pivot chosen for the first iteration
b3A1ft: Their pivot row correct including change of b.v. following their first iteration
b4M1: (ft) **All** values in one of the non-pivot rows correct **or** one of the non zero/one columns (*x, r, s* or value) correct following through their choice of pivot
b4A1: CAO for all values and row operations for **both** iterations - including all eight row operations stated correctly (ignore b.v. column for this mark)

c1M1: Their correct values stated for at least *P, x, y, z* from their 'optimal' iteration so there must be no negatives in the profit row. Two M marks in (b) must have been awarded – the numerical value of *P* must be explicitly stated and not as part of an equation
c1A1: CAO for all seven values explicitly stated

If pivoting on any other positive value for the first iteration then candidates can score in (b) and (c):

(b) M0A0M1A0 M1A1M0A0 (c) M1A0 (so max. of 4/10)

Question Number	Scheme	Marks
6. (a)	Initial flow = 57	B1 **(1)**
(b)	Value of the cut = 102	B1 **(1)**
(c)		M1 A1 **(2)**
(d)	e.g. SBEGT – 2; SBACT – 5; SBCT – 3; SBEDFGT – 1 e.g. SBCT – 7; SBEGT – 2; SBEDFGT – 1; SBACT – 1	M1A1A1A1 **(4)**
(e)	e.g.  **Or as above with AC(15), CB(0) and AB(9)**	M1 A1 **(2)**
(f)	The cut through CT, BT, ET, EG and FG has a value of 68 Value of the flow is 68 so by the max flow – min cut theorem flow is maximal	DB1 DB1 **(2)**
		12 marks

Question Number	Scheme	Marks

Notes for Question 6

a1B1: CAO

b1B1: CAO

c1M1: Two numbers on each arc and any four numbers correct
c1A1: CAO do give bod since they might well cross these numbers out (in attempting (d))

d1M1: One valid flow augmenting route found and any value stated
d1A1: A second correct flow route and any value stated
d2A1: Three correct flow routes with corresponding correct values
d3A1: CSO flow increased by 11 and no more

e1M1: Consistent flow pattern ≥ 61 (check each node). One number only per arc. No unnumbered arcs
e1A1: CAO showing flow of 68

f1DB1: Must have attempted (e) and scored at least M1A1 in (d) – at least one number on all but one arc, and either drawn or stated a cut. Cut may be drawn on any diagram. Note that the cut must separate source (S) from sink (T)
f2DB1: CSO – (e) must be fully correct (showing a correct flow of 68) and a correct cut (either stated or shown on any diagram). Must state the value of 68 in their answer and refer to max flow – min cut theorem – all four words

Question Number	Scheme	Marks
7. (a)		

Stage	State	Action	Dest.	Value
T-shirt	0	0	0	0
	1	1	0	55
	2	2	0	95
	3	3	0	180
	4	4	0	230
	5	5	0	290
Rugby	0	0	0	0
	1	1	0	65 + 0 = 65*
		0	1	0 + 55 = 55
	2	2	0	100 + 0 = 100
		1	1	65 + 55 = 120*
		0	2	0 + 95 = 95
	3	3	0	160 + 0 = 160
		2	1	100 + 55 = 155
		1	2	65 + 95 = 160
		0	3	0 + 180 = 180*
	4	4	0	245 + 0 = 245*
		3	1	160 + 55 = 215
		2	2	100 + 95 = 195
		1	3	65 + 180 = 245*
		0	4	0 + 230 = 230
	5	5	0	285 + 0 = 285
		4	1	245 + 55 = 300*
		3	2	160 + 95 = 255
		2	3	100 + 180 = 280
		1	4	65 + 230 = 295

M1 A1 (stage 1)

M1 A1 A1 (1st 4 states of stage 2)

M1 A1 A1 (Last 2 states of stage 2)

<table>
<tr><td>Question Number</td><td>Scheme</td><td>Marks</td></tr>
<tr><td></td><td>
<table>
<tr><td></td><td></td><td>0</td><td>5</td><td>0 + 290 = 290</td></tr>
<tr><td>Polo</td><td>5</td><td>5</td><td>0</td><td>310 + 0 = 310</td></tr>
<tr><td></td><td></td><td>4</td><td>1</td><td>225 + 65 = 290</td></tr>
<tr><td></td><td></td><td>3</td><td>2</td><td>175 + 120 = 295</td></tr>
<tr><td></td><td></td><td>2</td><td>3</td><td>110 + 180 = 290</td></tr>
<tr><td></td><td></td><td>1</td><td>4</td><td>70 + 245 = 315*</td></tr>
<tr><td></td><td></td><td>0</td><td>5</td><td>0 + 300 = 300</td></tr>
</table>
</td><td>M1 A1
(3rd stage)</td></tr>
<tr><td></td><td>Profit = (£) 315,000</td><td>A1 (11)</td></tr>
<tr><td>(b)</td><td>(T-shirt = 0,) Rugby = 4, Polo = 1 or T-shirt = 3, Rugby = 1, Polo = 1</td><td>B1 B1 (2)</td></tr>
<tr><td></td><td></td><td>13 marks</td></tr>
</table>

Notes for Question 7

a1M1: At least five rows for the first stage. Value column must contain the values of 55, 95, 180, 230 and 290. Ignore entries in all other columns and condone an error in one value only
a1A1: CAO for the first stage (all six rows) – **entries in all columns must be correct** – candidates may start with state 5 (rather than state 0) which is fine

a2M1: Second stage – my states 1, 2 and 3 (so at least 9 rows in the first half of the second stage or at least 20 non-zero rows). Value column must be complete with at least one value correct for each state – ignore entries in all other columns
a2A1: Value column for states 1, 2 and 3 correct for the second stage – ignore entries in all other columns and condone additional rows
a3A1: CAO for states 0, 1, 2 and 3 of the second stage (no additional rows for these four states) - **entries in all columns must be correct**

a3M1: Second stage – my states 4 and 5 (so at least 11 rows in the second half of the second stage or at least 20 non-zero rows). Value column must be complete with at least one value correct for each state – ignore entries in all other columns
a4A1: Value column for states 4 and 5 correct for the second stage – ignore entries in all other columns and condone additional rows
a5A1: CAO for states 4 and 5 of the second stage (no additional rows for these two states) - **entries in all columns must be correct**

If $9 \leq$ non-zero rows < 20 and it is unclear which rows relate to which state but there is one correct value from all five states then award the first M mark for this (2nd) stage

a4M1: At least 6 rows for the third stage. Value column must be complete with at least 3 values correct – ignore entries in all other columns
a6A1: CAO for third stage correct (no additional rows for this stage) - **entries in all columns must be correct**

Question Number	Scheme	Marks
	a7A1: CAO – must have earned all previous M marks **b1B1:** One correct allocation (dependent on at least three M marks awarded in (a)) **b2B1:** Both allocations correct (dependent on at least three M marks awarded in (a))	

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